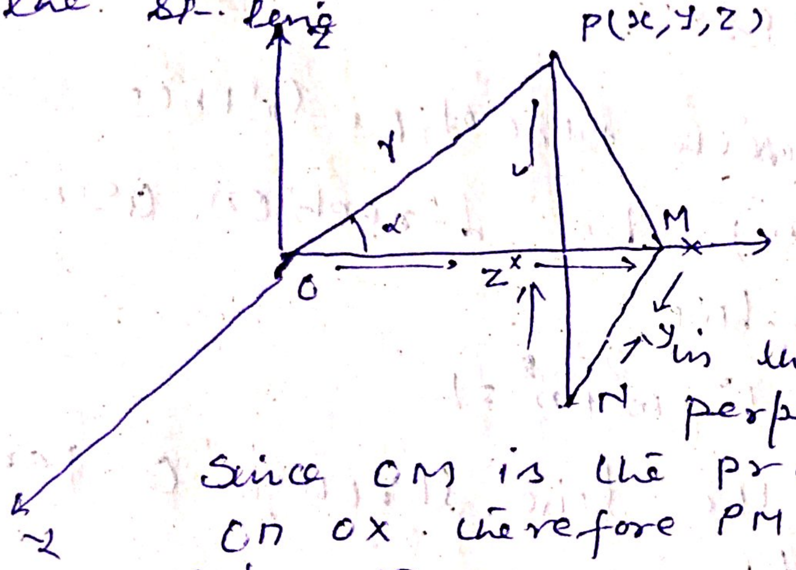
Deptt — Mathematics
Topic — Direction Cosines
(Analytical Geometry 3-D).
class - B.Sc (Part-I)
Time — 1.00 P.M to 1.45 PM
Date — 28-07-2020

By - Dr. Shree Nath Sharma

Study Material → Direction cosines of a st. line are usually denoted by, l, m, n

If a st. line makes angle α, β, γ with the +ve direction of x-axis, y-axis and z-axis respectively then $l = \cos\alpha$ $m = \cos\beta$ $n = \cos\gamma$ are called Direction-cosines of the st. line



Here PN is perpendicular to the plane xy and NM in the plane XOY perpendicular to OX

Since OM is the projection of OP on OX therefore PM is also perpendicular to OX.

Let (x, y, z) be the co-ordinate of P and $OP = r$

$\therefore OM = x \quad NM = y, \; NP = z$

Also $\angle POM = \alpha$

From the right-angled $\triangle OPM$

$$\cos\alpha = \frac{OM}{OP} = \frac{x}{r}$$

$\therefore x = r\cos\alpha = lr$

Similarly $y = r\cos\beta = mr$ and $z = r\cos\gamma = nr$

Thus we see

(lr, mr, nr) will give us the co-ordinates of a point on the line OP which is at a distance r from $(0,0,0)$

If $P(x,y,z)$ is any point on the line through $A(x_1,y_1,z_1)$ whose direction cosines are $\cos\alpha, \cos\beta, \cos\gamma$ such that $AP = r$

then $\frac{x-x_1}{r} = \cos\alpha$, $\frac{y-y_1}{r} = \cos\beta$, and $\frac{z-z_1}{r} = \cos\gamma$

$\Rightarrow x = x_1 + r\cos\alpha$

$y = y_1 + r\cos\beta$

$z = z_1 + r\cos\gamma$.

* Relation between direction cosines →

If (l, m, n) be direction cosine of any st. line

then $l^2 + m^2 + n^2 = 1$

Let OP be the line through O parallel to given line.

thus direction cosine of OP are l, m, n

$OP^2 = r^2 = x^2 + y^2 + z^2$ $\quad\because x = lr,\ y = mr,\ z = nr$.

$= l^2r^2 + m^2r^2 + n^2r^2$

$= (l^2 + m^2 + n^2)r^2$

$\therefore l^2 + m^2 + n^2 = 1$

Direction ratio → If a, b, c be three Numbers proportional to the actual direction Co-sines say (l, m, n) then we have

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = k \text{ (say)}$$

$L = ak \quad m = bk \quad n = ck$

$\Rightarrow l^2 = a^2k^2 \quad m^2 = b^2k^2, \quad n = c^2k^2$

But $l^2 + m^2 + n^2 = 1$

$\Rightarrow (a^2 + b^2 + c^2) k^2 = 1$

$$\Rightarrow k^2 = \frac{1}{a^2+b^2+c^2}$$

$$\Rightarrow k = \pm \frac{1}{\sqrt{a^2+b^2+c^2}}$$

$$\text{Now } l = ak = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m = bk = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n = ck = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Numbers which are proportional to the actual Direction cosines are called Direction Numbers.

Direction cosines of a line Joining (x_1, y_1, z_1) and (x_2, y_2, z_2) are Proportional to $x_2 - x_1$,
$y_2 - y_1$
$z_2 - z_1$

Angle Between two Lines :- 28-07-2…

(i) $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

$\Rightarrow \theta = \cos^{-1}(l_1 l_2 + m_1 m_2 + n_1 n_2)$

Where (l_1, m_1, n_1) and (l_2, m_2, n_2) are direction cosines of two given St. l…

(ii) $\sin\theta = \sqrt{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l…}$

Applying $\sin^2\theta = 1 - \cos^2\theta$

and hence $\sin\theta = \sqrt{1 - \cos^2\theta}$

Special Cases → (1) the given lines are perpendicular if

$\theta = 90$

$\Rightarrow \cos\theta = \cos 90 = 0$

$\Rightarrow l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$.

(2) the given lines are parallel if $\theta = 0$

$\Rightarrow \sin\theta = \sin 0 = 0$

$\Rightarrow (m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m…$

it is possible only when

$(m_1 n_2 - m_2 n_1)^2 = 0 \quad (n_1 l_2 - n_2 l_1)^2 = 0$

$(l_1 m_2 - l_2 m_1)^2 = 0$

i.e $\dfrac{m_1}{m_2} = \dfrac{n_1}{n_2} = \dfrac{l_1}{l_2}$

(3) $(x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n$ is projection of a line whose direction cosines are l, m, n